

Hong Kong Mathematics Olympiad (2008 / 2009)

Final Event 1 (Individual)

香港數學競賽 (2008 / 2009)

決賽項目 1 (個人)

除非特別聲明，答案須用數字表達，並化至最簡。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

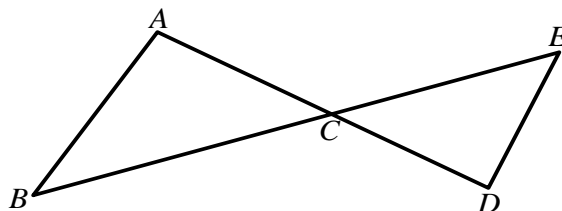
1. 設 a 、 b 、 c 及 d 為方程 $x^4 - 15x^2 + 56 = 0$ 相異的根。若 $R = a^2 + b^2 + c^2 + d^2$ ，求 R 的值。

Let a , b , c and d be the distinct roots of the equation $x^4 - 15x^2 + 56 = 0$. If $R = a^2 + b^2 + c^2 + d^2$, find the value of R .



2. 如圖一， AD 及 BE 為直線且 $AB = AC$ 及 $AB \parallel ED$ 。若 $\angle ABC = R^\circ$ 及 $\angle ADE = S^\circ$ ，求 S 的值。

In Figure 1, AD and BE are straight lines with $AB = AC$ and $AB \parallel ED$. If $\angle ABC = R^\circ$ and $\angle ADE = S^\circ$, find the value of S .



圖一
Figure 1



3. 設 $F = 1 + 2 + 2^2 + 2^3 + \cdots + 2^s$ 及 $T = \sqrt{\frac{\log(1+F)}{\log 2}}$ ，求 T 的值。

Let $F = 1 + 2 + 2^2 + 2^3 + \cdots + 2^s$ and $T = \sqrt{\frac{\log(1+F)}{\log 2}}$. Find the value of T .



4. 設 $f(x)$ 是一個函數使得對所有整數 $n \geq 6$ 時， $f(n) = (n-1)f(n-1)$ 及 $f(n) \neq 0$ 。若

$$U = \frac{f(T)}{(T-1)f(T-3)}, \text{ 求 } U \text{ 的值。}$$

Let $f(x)$ be a function such that $f(n) = (n-1)f(n-1)$ and $f(n) \neq 0$ hold for all integers $n \geq 6$.

If $U = \frac{f(T)}{(T-1)f(T-3)}$, find the value of U .



Hong Kong Mathematics Olympiad (2008 / 2009)

Final Event 2 (Individual)

香港數學競賽 (2008 / 2009)

決賽項目 2 (個人)

除非特別聲明，答案須用數字表達，並化至最簡。

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1. 設 $[x]$ 是不超過 x 的最大整數。若 $a = \left[(\sqrt{3} - \sqrt{2})^{2009} \right] + 16$ ，求 a 的值。

Let $[x]$ be the largest integer not greater than x . If $a = \left[(\sqrt{3} - \sqrt{2})^{2009} \right] + 16$, find the value of a .

2. 在坐標平面上，若 x -軸、 y -軸與直線 $3x + ay = 12$ 所圍成三角形的面積是 b 平方單位，求 b 的值。

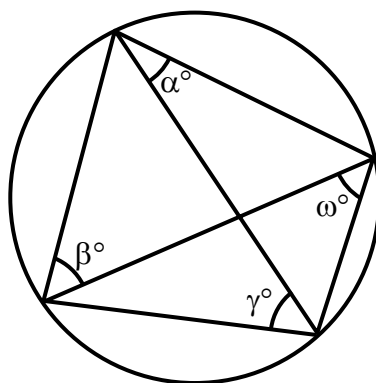
In the coordinate plane, if the area of the triangle formed by the x -axis, y -axis and the line $3x + ay = 12$ is b square units, find the value of b .

3. 已知 $x - \frac{1}{x} = 2b$ 及 $x^3 - \frac{1}{x^3} = c$ ，求 c 的值。

Given that $x - \frac{1}{x} = 2b$ and $x^3 - \frac{1}{x^3} = c$, find the value of c .

4. 如圖一， $\alpha = c$ ， $\beta = 43$ ， $\gamma = 59$ 及 $\omega = d$ ，求 d 的值。

In Figure 1, $\alpha = c$, $\beta = 43$, $\gamma = 59$ and $\omega = d$, find the value of d .



圖一

Figure 1



Hong Kong Mathematics Olympiad (2008 / 2009)

Final Event 3 (Individual)

香港數學競賽 (2008 / 2009)

決賽項目 3 (個人)

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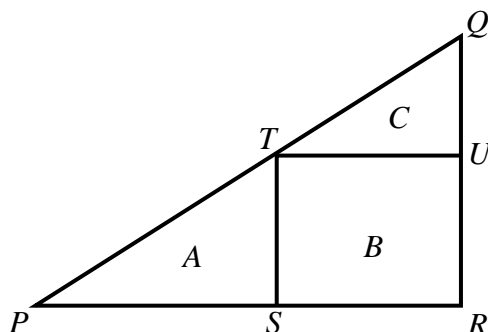
1. 已知 $\frac{4}{\sqrt{6}+\sqrt{2}} - \frac{1}{\sqrt{3}+\sqrt{2}} = \sqrt{a} - \sqrt{b}$ ，其中 a, b 為正整數。若 $m = a - b$ ，求 m 的值。

Given that $\frac{4}{\sqrt{6}+\sqrt{2}} - \frac{1}{\sqrt{3}+\sqrt{2}} = \sqrt{a} - \sqrt{b}$, where a, b are positive integers. If $m = a - b$, find the value of m .



2. 如圖一， PQR 為直角三角形及 $RSTU$ 為矩形。設 A 、 B 及 C 是相對圖形的面積。若 $A : B = m : 2$ 及 $A : C = n : 1$ ，求 n 的值。

In Figure 1, PQR is a right-angled triangle and $RSTU$ is a rectangle. Let A , B and C be the areas of the corresponding regions. If $A : B = m : 2$ and $A : C = n : 1$, find the value of n .



圖一

Figure 1



3. 設 x_1 、 x_2 、 x_3 、 x_4 為實數及 $x_1 \neq x_2$ 。若 $(x_1 + x_3)(x_1 + x_4) = (x_2 + x_3)(x_2 + x_4) = n - 10$ 及 $p = (x_1 + x_3)(x_2 + x_3) + (x_1 + x_4)(x_2 + x_4)$ ，求 p 的值。

Let x_1, x_2, x_3, x_4 be real numbers and $x_1 \neq x_2$. If $(x_1 + x_3)(x_1 + x_4) = (x_2 + x_3)(x_2 + x_4) = n - 10$ and $p = (x_1 + x_3)(x_2 + x_3) + (x_1 + x_4)(x_2 + x_4)$, find the value of p .

4. 已知某校學生人數是 7 的倍數且不少於 1000。若學生人數被 $p + 1$ 、 $p + 2$ 及 $p + 3$ 除後的餘數均是 1。設學生人數的最小可能值為 q ，求 q 。

The total number of students in a school is a multiple of 7 and not less than 1000. Given that the same remainder 1 will be obtained when the number of students is divided by $p + 1$, $p + 2$ and $p + 3$. Let q be the least of the possible numbers of students in the school, find q .

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Final Event 4 (Individual)

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決賽項目 4 (個人)

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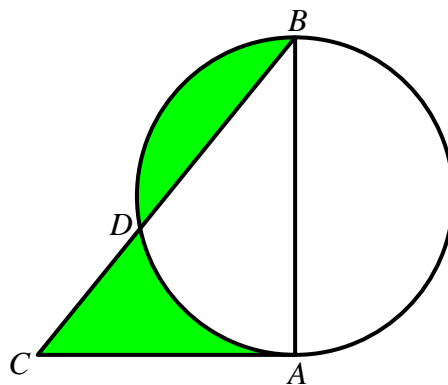
1. 已知 $x_0^2 + x_0 - 1 = 0$ 。若 $m = x_0^3 + 2x_0^2 + 2$ ，求 m 的值。

Given that $x_0^2 + x_0 - 1 = 0$. If $m = x_0^3 + 2x_0^2 + 2$, find the value of m .



2. 如圖一， $\triangle BAC$ 是一直角三角形， $AB = AC = m$ cm。已知直徑為 AB 的圓與 BC 相交於 D 且陰影部分的面積是 n cm²，求 n 的值。

In Figure 1, $\triangle BAC$ is a right-angled triangle, $AB = AC = m$ cm. Suppose that the circle with diameter AB intersects the line BC at D , and the total area of the shaded region is n cm². Find the value of n .



圖一

Figure 1



3. 已知 $p = 4n \left(\frac{1}{2^{2009}} \right)^{\log(1)}$ ，求 p 的值。

Given that $p = 4n \left(\frac{1}{2^{2009}} \right)^{\log(1)}$, find the value of p .

4. 設 x 及 y 為實數並滿足方程 $(x - \sqrt{p})^2 + (y - \sqrt{p})^2 = 1$ 。若 $k = \frac{y}{x-3}$ 及 q 是 k^2 的最小可能值，求 q 。

Let x and y be real numbers satisfying the equation $(x - \sqrt{p})^2 + (y - \sqrt{p})^2 = 1$. If $k = \frac{y}{x-3}$ and q is the least of the possible values of k^2 , find q .